

Plank's law.

In 1901 Plank introduced entirely new ideas to explain the distribution of energy due among the various of the cavity radiation, behave as oscillators. He assumed that the atoms of the walls of the cavity radiator behave as oscillators. These oscillators emit electro-magnetic radiant energy in to the cavity and also absorb the same from it and maintain an equilibrium state. Plank made two revolutionary assumptions regarding these atomic oscillators.

① An oscillator can have only discrete energy given by

$$E = n h \nu$$

where ν is the frequency of the oscillator, h is the constant known as Plank's constant, n is an integer known as "quantum number". This means that the oscillator can have only the energies $h\nu, 2h\nu, 3h\nu \dots$ and not any energy in between. In other words the energy of the oscillator is quantised.

② The oscillators do not absorb or emit energy continuously but only in jumps. That is an oscillator emits or absorbs packets of energy, each packet carrying an amount of energy $h\nu$.

$$\Delta E = (\Delta n) h \nu, \Delta n = 1, 2, 3 \dots$$

Now let us calculate the average energy of a planks oscillator of frequency ν . The relative probability that an oscillator has the energy $h\nu$ at temperature T is given by the Boltzmann factor $e^{-h\nu/KT}$.

Let $N_0, N_1, N_2 \dots N_r$ be the number of oscillators having the energies $0, h\nu, 2h\nu, \dots rh\nu$, respectively

(P.T.O)

from page - 1

$$\text{Then we have } N_r = N_0 e^{-hv/kT}$$

$$\begin{aligned} \therefore N &= N_0 + N_1 + N_2 + N_3 + \dots \\ &= N_0 + N_0 \frac{e^{-hv/kT}}{1-e^{-hv/kT}} + N_0 e^{-2hv/kT} + \dots \\ &= N_0 \left(1 + e^{-hv/kT} + e^{-2hv/kT} + \dots\right) \\ &= \frac{N_0}{1 - e^{-hv/kT}} \end{aligned} \quad \rightarrow \textcircled{1}$$

\therefore Total energy of the oscillator is given by -

$$E = (N_0 \times 0) + (N_1 + hv) + N_2 \times 2hv + (N_3 \times 3hv) + \dots$$

After solving this equation, we get -

$$E = N_0 e^{-hv/kT} \frac{hv}{(1 - e^{-hv/kT})^2} \rightarrow \textcircled{2}$$

Dividing eqn. $\textcircled{2}$ by eqn. $\textcircled{1}$ we get, the average energy of an oscillator as given by

$$\bar{E} = \frac{E}{N} = N_0 e^{-hv/kT} \frac{hv}{(1 - e^{-hv/kT})^2} \cancel{\frac{hv}{1 - e^{-hv/kT}}} = \frac{N_0}{1 - e^{-hv/kT}}$$

$$\text{or } \bar{E} = \frac{E}{N} = \frac{hv}{e^{-hv/kT} - 1} \rightarrow \textcircled{3}$$

Now $E \propto v^3$ the density of radiation of frequencies between v and $v + dv$ is related to the average energy of an oscillator emitting v radiation, given by -

$$E v dv = \frac{8\pi v^2}{c^3} (dv) \bar{E}$$

$$\text{or, } E v dv = \frac{8\pi v^2}{c^3} (dv) \frac{hv}{e^{-hv/kT} - 1}$$

$$\text{or, } E v dv = \frac{8\pi h}{c^3} \frac{(dv)}{v^3} \frac{v^3 dv}{e^{-hv/kT} - 1} \rightarrow \textcircled{4}$$

This is Planck's radiation formula in terms of frequency to express in terms of wavelength we have $v = \frac{c}{\lambda}$.